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Analytical and numerical heat transfer in cooling panels

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Abstract—A heat transfer analysis of cooling panels is presented, based on one-dimensional analytical solutions in the directions normal and parallel to the tubes of the cooling panel. A two-dimensional finite-difference solution is also obtained, which is in very good agreement with the one-dimensional analytical solution, especially for thin panels. Therefore, the one-dimensional solution, which is very simple to apply, is recommended for calculations in practice.

INTRODUCTION

ACCORDING to the method of cooling buildings using cooling panels [1], the heat produced within or inserted into a room is absorbed by a cold fluid flowing through tubes imbedded in the ceiling of the room, as shown in Fig. 1. In new buildings, the ceiling is constructed to form the cooling panel, while in the case of old buildings a metal sheet incorporating cooling tubes is placed onto the lower surface of the ceiling.

The objectives of the present study are: (a) to calculate the temperature field on the cooling panel; (b) to calculate the heat absorbed per unit area of the panel; and (c) to define quantities characterizing the panel efficiency. For this purpose, one-dimensional analytical solutions are obtained in the direction x, normal to the tubes, as well as in the direction z of the flow (Fig. 1). For the purpose of this analysis use is being made of the theory of flat plate solar collectors [2], which have some similarities to cooling panels.

In order to evaluate the accuracy of the one-dimensional analytical solution, which is based on certain assumptions and approximations, the heat transfer problem is also solved as a two-dimensional one using the finite-difference method. The results of the two methods are in very good agreement, especially for thin panels, and therefore use of the analytical solution for practical applications is recommended, as it is simpler.

ANALYTICAL SOLUTION

Temperature variation between tubes

A Cartesian coordinate system x, y, z is considered, as shown in Fig. 1, i.e. coordinates x and y lie on the plane normal to the tubes, and z is the direction of the flow. It is assumed that the temperature variation along the thickness w of the panel (i.e. in the y direction) is negligible. It is also assumed, temporarily, that the temperature variation in the flow direction is negligible. As illustrated in Fig. 2, an energy balance on an element of width Δx and unit length in the flow direction yields

$$h\Delta x(T_r - T) + \left[-kw\frac{dT}{dx}\right]_x - \left[-kw\frac{dT}{dx}\right]_{x+\Delta x} = 0 \tag{1}$$

where h is the heat transfer coefficient of the lower surface of the panel, T_r the room temperature, $T(x)$ the temperature of the panel and k its thermal conductivity. Considering that $[dT/dx]_{x+dx} = [dT/dx]_x + d(dT/dx)$, equation (1) gives after division by $kw\Delta x$ and replacement of Δx by dx:

$$\frac{d^2T}{dx^2} = m^2(T - T_r) \tag{2}$$

where

$$m^2 = h/(kw). \tag{3}$$

The boundary conditions for equation (2) are

$$\left[\frac{dT}{dx}\right]_{x=0} = 0, \qquad [T]_{x=(S-D_o)/2} = T_0 \tag{4}$$

where T_0 is the temperature of the panel at the location of the tube, i.e. from $x = (S-D_o)/2$ to $x = (S+D_o)/2$. Integration of equation (2) with the above boundary conditions yields the temperature distribution between the tubes, i.e.

$$\frac{T - T_r}{T_0 - T_r} = \frac{\cos h(mx)}{\cos h[m(S-D_o)/2]}. \tag{5}$$

Figure 3 shows the temperature variation between the tubes according to equation (5) for $h = 10$ W m^{-2} °C^{-1}, $T_r = 24$°C, $k = 1.4$ W m^{-1} °C^{-1}, $w = 0.04$ m, $T_0 = 12$°C, $D_o = 0.02$ m and $S = 0.3$ m.

NOMENCLATURE

A coefficient in finite-difference equation (34)
A_p panel area [m^2]
c_p specific heat [J kg^{-1} K^{-1}]
D_o, D_i outside and inside tube diameters, respectively [m]
F factor given by equation (9) (dimensionless)
F_1 panel efficiency factor (dimensionless)
F_2 panel flow factor (dimensionless)
F_c panel cooling efficiency (dimensionless)
G dimensionless panel mass flow rate
h, h_f, h_u heat transfer coefficients of the panel lower surface, of the inside tube surface and of the upper surface of the ceiling, respectively [W m^{-2} K^{-1}]
k thermal conductivity [W m^{-1} K^{-1}]
L panel length in flow direction [m]
m quantity defined by equation (3) [m^{-1}]
$\dot{m}$ mass flow rate [kg s^{-1}]
n number of tubes in the panel
q^* heat per unit length in flow direction [W m^{-1}]
q heat per unit area of the panel [W m^{-2}]
S distance between the axes of two adjacent tubes [m]
T local panel temperature [°C]
T_0 panel temperature at the location of a tube [°C]
T_f local fluid temperature [°C]
$T_{f,i}$, $T_{f,o}$, $T_{f,m}$ inlet, outlet and mean fluid temperatures [°C]
$T_{p,m}$ mean panel temperature [°C]
T_r room air temperature [°C]
T_u air temperature above the upper surface of a ceiling [°C]
w panel thickness [m]
w_0 total thickness of a ceiling [m]
x, y, z Cartesian coordinates
Δx, Δz elements in the x and z directions, respectively [m].

Subscripts
P, E, W, N, S refer to the typical node and its four neighbours of the finite-difference grid.

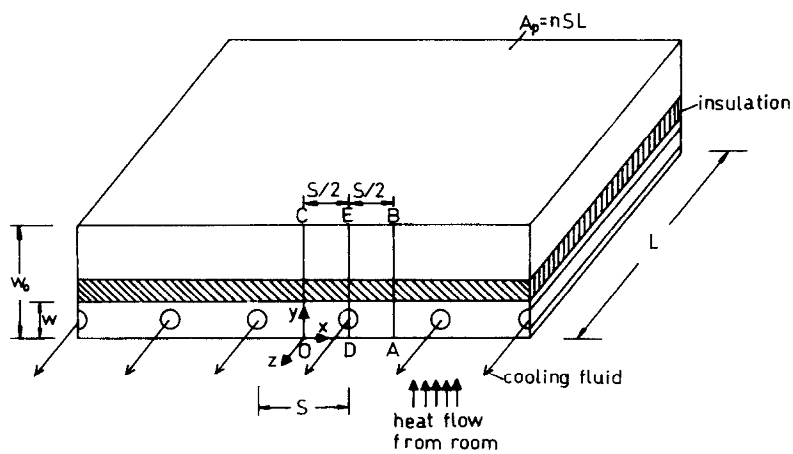

FIG. 1. Ceiling forming a cooling panel.

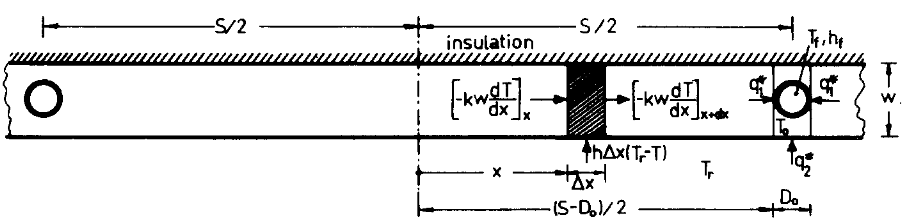

FIG. 2. Energy balance on an element Δx of the cooling panel.

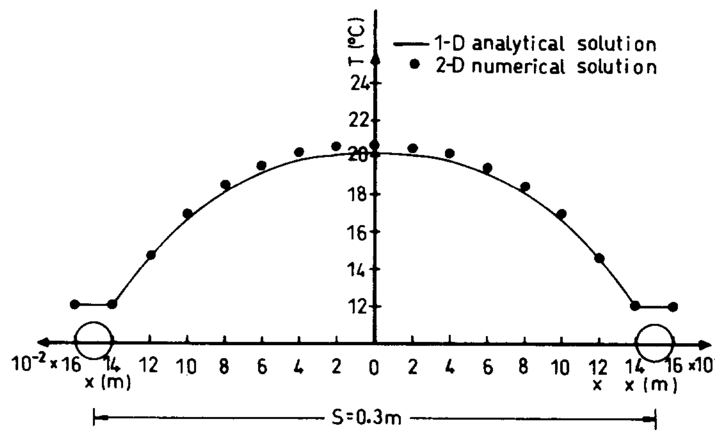

FIG. 3. One-dimensional analytical and two-dimensional numerical prediction of the temperature variation between the tubes of the panel, under the conditions mentioned in the text.

Heat absorbed by the panel

With reference to Fig. 2, the heat q^* conducted to each tube per unit length in the flow direction is

$$q^* = q_1^* + q_1^* + q_2^* \tag{6}$$

where q_2^* is the heat inserted through the region of the panel just below the tube, i.e.

$$q_2^* = D_o h(T_r - T_0) \tag{7}$$

and q_1^* is the heat flow in the x direction, calculated by differentiation of equation (5), i.e.

$$q_1^* = -kw\left[\frac{dT}{dx}\right]_{x=(S-D_o)/2} = 0.5h(T_r - T_0)(S - D_o)F \tag{8}$$

where

$$F = \frac{\tanh[m(S-D_o)/2]}{m(S-D_o)/2}. \tag{9}$$

Substitution from equations (7) and (8) into equation (6) yields

$$q^* = h(T_r - T_0)[D_o + (S - D_o)F]. \tag{10}$$

Because the tube wall thickness is small and its thermal conductivity high, the thermal resistance of the tube may be neglected and the heat flow q^* to the fluid may be expressed as

$$q^* = \pi D_i h_f (T_0 - T_f) \tag{11}$$

where T_f is the local fluid temperature, D_i the inside tube diameter and h_f the heat transfer coefficient on the inside surface of the tube. Substitution of T_0 from equation (11) into equation (10) and solution for q^* yields

$$q^* = SF_1 h(T_r - T_f) \tag{12}$$

where

$$F_1 = \frac{\frac{1}{h}}{S\left[\frac{1}{h[D_o + (S-D_o)F]} + \frac{1}{\pi D_i h_f}\right]}. \tag{13}$$

F_1 can be considered as the 'panel efficiency factor'. Equation (12) suggests that F_1 expresses the ratio of the actual cooling effect of the panel, to the cooling effect that would result if the panel surface were at the local fluid temperature T_f. Equation (13) suggests that F_1 expresses the ratio of the heat transfer resistance from the panel surface to the room air, to the heat transfer resistance from the fluid to the room air.

The heat q absorbed per unit area of the panel may be calculated from equation (12), i.e.

$$q = q^*/S = F_1 h(T_r - T_f). \tag{14}$$

Figure 4 shows q in terms of T_f with S as a parameter, calculated according to equation (14) for the following values of the remaining parameters: $h = 10$ W m^{-2} °C^{-1}, $T_r = 24$°C, $D_o = 0.020$ m, $D_i = 0.018$ m, $h_f = 3000$ W m^{-2} °C^{-1}, $k = 1.4$ W m^{-1} °C^{-1} and $w = 0.04$ m.

Temperature distribution in flow direction

The cooling fluid enters the panel at temperature $T_{f,i}$ and leaves it at $T_{f,o} > T_{f,i}$. With reference to Fig. 5, which shows a single tube of the panel, an energy balance on an element Δz of the fluid yields

$$\left[\frac{\dot{m}}{n}c_p T_f\right]_z - \left[\frac{\dot{m}}{n}c_p T_f\right]_{z+\Delta z} + q^*\Delta z = 0. \tag{15}$$

By considering that $[T_f]_{z+\Delta z} = [T_f]_z + dT_f$, substituting q^* from equation (12) and replacing Δz by dz, equation (15) becomes

$$\dot{m}c_p\frac{dT_f}{dz} - nSF_1 h(T_r - T_f) = 0. \tag{16}$$

Integration of the above equation with boundary con-

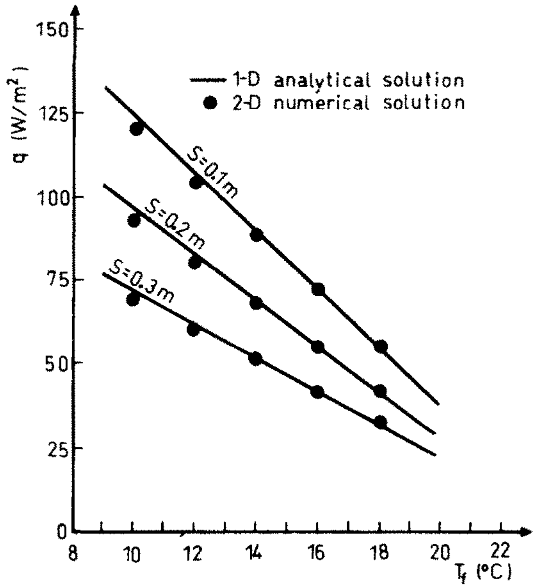

FIG. 4. Analytical and numerical prediction of the heat, q, absorbed per unit area of the panel in terms of the fluid temperature, T_f, with tube spacing, S, as a parameter, under the conditions mentioned in the text.

dition $T_f = T_{f,i}$ at $z = 0$, yields the temperature distribution in the flow direction, i.e.

$$\frac{T_f - T_r}{T_{f,i} - T_r} = \exp(-hnSF_1 z/\dot{m}c_p). \quad (17)$$

If the panel length in the flow direction is L, then the outlet fluid temperature, $T_{f,o}$ may be calculated from equation (17) for $z = L$, i.e.

$$\frac{T_{f,o} - T_r}{T_{f,i} - T_r} = \exp(-A_p h F_1/\dot{m}c_p) \quad (18)$$

where

$$A_p = nSL \quad (19)$$

is the panel area.

Panel cooling efficiency

The 'panel cooling efficiency' F_c can be defined as the ratio of the actual cooling effect of the panel to the cooling effect that would result if the panel surface were at the inlet fluid temperature, i.e.

$$F_c = \frac{\dot{m}c_p(T_{f,o} - T_{f,i})}{A_p h(T_r - T_{f,i})}. \quad (20)$$

By using equation (18), the above equation becomes

$$F_c = \frac{\dot{m}c_p}{A_p h}[1 - \exp(-A_p h F_1/\dot{m}c_p)] = F_1 F_2 \quad (21)$$

where

$$F_2 = \frac{\dot{m}c_p}{A_p h F_1}[1 - \exp(-A_p h F_1/\dot{m}c_p)]$$
$$= G[1 - \exp(-1/G)]. \quad (22)$$

Quantity F_2 is a function of a single variable, i.e. the dimensionless panel mass flow rate

$$G = \dot{m}c_p/A_p h F_1 \quad (23)$$

and may be named 'panel flow factor'.

By using equation (20), the heat q absorber per unit area of the panel may be expressed as

$$q = F_c h(T_r - T_{f,i}). \quad (24)$$

The above equation is more useful than equation (14) because it allows calculation of the heat q in terms of the known fluid inlet temperature $T_{f,i}$.

Mean fluid and panel temperatures

The mean fluid temperature

$$T_{f,m} = \frac{1}{L}\int_0^L T_f \, dy \quad (25)$$

can be calculated by substituting T_f from equation (17) into the above equation and then integrating. Using also equations (21) and (24) the following expression is found for the mean fluid temperature:

$$T_{f,m} = T_{f,i} + \frac{q}{hF_c}(1 - F_2). \quad (26)$$

The heat q absorbed per unit area of the panel, which is given by equations (14) or (24), may also be expressed in terms of the mean panel surface temperature $T_{p,m}$ as:

$$q = h(T_r - T_{p,m}). \quad (27)$$

By combining the above equation with equation (24), the following expression is derived for the mean panel temperature

$$T_{p,m} = T_{f,i} + \frac{q}{hF_c}(1 - F_c). \quad (28)$$

TWO-DIMENSIONAL NUMERICAL SOLUTION

With reference to Fig. 1, assuming negligible variation of temperature in the flow direction z, the steady-state two-dimensional heat conduction equa-

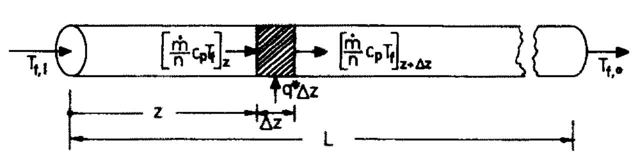

FIG. 5. Energy balance on an element Δz of the cooling fluid.

tion on the plane x–y normal to the tubes may be written as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=0 \qquad (29)$$

where the thermal conductivity k takes a different value in each of the various layers of the ceiling. Differential equation (29) may be solved by the finite-difference method within the unit of symmetry OABC (i.e. for $0 \leqslant x \leqslant S$, $0 \leqslant y \leqslant w_0$) or ODEC (i.e. for $0 \leqslant x \leqslant S/2$, $0 \leqslant y \leqslant w_0$), with the following boundary conditions.

On the lower surface OA, the heat flow $q(x,0)$ is prescribed, i.e.

$$q(x,0) = h[T_r - T(x,0)] \qquad (30)$$

where h is the heat transfer coefficient of the lower surface of the panel and T_r is the room air temperature.

On the upper surface CB, the heat flow $q(x, w_0)$ is prescribed as

$$q(x,w_0) = h_u[T_u - T(x,w_0)] \qquad (31)$$

where h_u is the heat transfer coefficient of the upper surface of the ceiling and T_u is the air temperature above the ceiling, which is considered internal, i.e. it separates two storeys. The problem of external ceilings, which is transient owing to the time-dependent T_u and the time-dependent incident solar radiation, is examined elsewhere [3].

On the inside surface of the tube, the heat flow $q(x,y)$ is prescribed as

$$q(x,y) = h_f[T(x,y) - T_f] \qquad (32)$$

where $T(x,y)$ and h_f are the temperature and the heat transfer coefficient of the inside surface of the tube, respectively, and T_f is the fluid temperature.

On boundaries OC and AB, which are planes of symmetry, the following boundary conditions are imposed:

$$\frac{\partial T}{\partial x}(0,y) = 0, \qquad \frac{\partial T}{\partial x}(S,y) = 0. \qquad (33)$$

Solution of differential equation (29) is obtained within the domain OABC of Fig. 1 by employing a usual finite-difference procedure (see, for example [4]) suitably modified so as to incorporate boundary conditions (30)–(33). Briefly, a Cartesian grid composed of coordinate lines x and y is imposed on the solution domain, with the tube periphery approximated by straight lines. Integration of differential equation (29) over each control-volume of the grid, yields finite-difference equations of the general form

$$A_P T_P = A_E T_E + A_W T_W + A_N T_N + A_S T_S \qquad (34)$$

where As are known coefficients and the subscripts refer to the typical node P and its four neighbours, E, W, N, S. The set of finite-difference equations (34) for all nodes P, combined with similar finite-difference

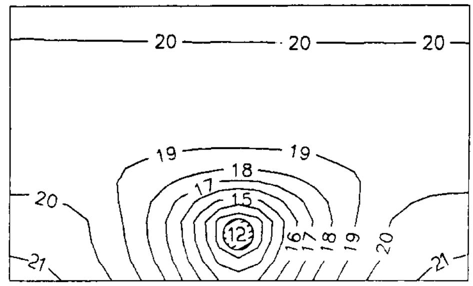

FIG. 6. Predicted temperature contours on the cross-section of a cooling panel, under the conditions mentioned in the text.

equations for the boundary control-volumes, is solved by employing usual techniques to give the temperature field. An example of the results is given in Fig. 6, which shows the predicted temperature contours for a ceiling composed of three layers of thicknesses (from the lower to the upper) $w = 0.04$ m, 0.04 m and 0.10 m with corresponding thermal conductivities $k = 1.4$ W m^{-1} °C^{-1}, 0.036 W m^{-1} °C^{-1} and 2.03 W m^{-1} °C^{-1}. The remaining parameters are fixed to the values $h = h_u = 10$ W m^{-2} °C^{-1}, $T_r = 24$°C, $h_f = 3000$ W m^{-2} °C^{-1}, $T_f = 12$°C, $D_o = 0.020$ m, $D_i = 0.018$ m and $S = 0.3$ m.

The predicted temperature variation along the x-direction line passing through the centers of the tubes for the same case as above, is shown in Fig. 3, together with the one-dimensional analytical solution (i.e. equation (5)). The agreement is very good.

Under the same conditions, Fig. 4 shows the predicted heat q absorbed per unit area of the panel, in terms of T_f with S as a parameter, together with the analytical solution (i.e. equation (14)). The agreement of the two solutions is very good.

CONCLUSION

Analytical solutions, expressed by equations (5) and (17), have been obtained for the temperature distributions between and along the tubes of a cooling panel, respectively.

Based on the above solutions, equations (14) and (24) have been derived, which express the heat absorbed per unit area of the cooling panel.

Quantities expressing the performance of a cooling panel have been introduced, i.e. the panel efficiency factor, F_1, the panel flow factor, F_2, and the panel cooling efficiency, F_c.

A two-dimensional finite-difference solution of the problem has also been obtained. Comparisons with the one-dimensional analytical solution showed very good agreement, especially for thin panels, as illustrated in the examples of Figs. 3 and 4. Therefore, the one-dimensional analytical solution, which is very simple to apply, is recommended for calculations in practice.

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TRANSFERT THERMIQUE ANALYTIQUE ET NUMERIQUE DANS LES PANNEAUX REFRIGERANTS

Résumé—Une analyse du transfert thermique des panneaux réfrigérants est présentée à partir des solutions analytiques unidirectionnelles dans les directions normale et parallèle aux tubes du panneau. On obtient aussi une solution bidimensionnelle aux différences finies qui est en bon accord avec la solution analytique unidirectionnelle, spécialement pour les panneaux minces. La solution unidirectionnelle qui est d'application très simple est recommandée par des calculs pratiques.

ANALYTISCHE UND NUMERISCHE BESTIMMUNG DES WÄRMETRANSPORTS IN KÜHLREGISTERN

Zusammenfassung—Der Wärmetransport in Kühlregistern wird untersucht. Dies geschieht auf der Grundlage eindimensionaler analytischer Lösungen senkrecht und parallel zu den Rohren des Kühlregisters. Zusätzlich wird die zweidimensionale Lösung einer Finite-Differenzen-Methode vorgestellt, die besonders für dünne Register sehr gut mit der eindimensionalen analytischen Lösung übereinstimmt. Deshalb wird die in ihrer Anwendung sehr einfache eindimensionale analytische Lösung zur praktischen Anwendung vorgeschlagen.

АНАЛИТИЧЕСКОЕ И ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ ТЕПЛОПЕРЕНОСА В ОХЛАЖДАЕМЫХ ПАНЕЛЯХ

Аннотация—Дан анализ теплопереноса в охлаждаемых панелях на основе одномерных аналитических решений в направлениях, перпендикулярном и параллельном трубам паиели. Конечно-разностным методом получено двумерное решение, результаты которого очень хорошо согласуются с результатами одномерного аналитического решения, особенно в случае тонких панелей. Поэтому для практических расчетов рекомендуется простое в использовании одномерное решение.